B.Sc. part-I (Sub) Set theory 01-07-2020

Problems based on Cartesian product

Q.1. If A, B be two sets, prove that

$$(A\times B)' = (A'\times B) \cup (A\times B') \cup (A'\times B')$$

Soln: Let $(x,y) \in (A\times B)'$ be arbitrary. Then

$(x,y) \in (A\times B)'$

$\Rightarrow (x,y) \notin (A\times B)$

$\Rightarrow (x\notin A \wedge y\in B) \vee (x\in A \wedge y\notin B) \vee (x\notin A \wedge y\notin B)$

$\Rightarrow (x\notin A \wedge y\in B) \vee (x\in A \wedge y\in B') \vee (x\in A' \wedge y\in B')$

$\Rightarrow (x\in A' \wedge y\in B) \vee (x\in A \wedge y\in B') \vee (x\in A' \wedge y\in B')$

$\Rightarrow (x,y) \in (A'\times B) \vee (x,y) \in (A\times B') \vee (x,y) \in (A'\times B')$

$\Rightarrow (x,y) \in (A'\times B) \cup (A\times B') \cup (A'\times B')$

$\Rightarrow (x,y) \in (A'\times B) \cup (A\times B') \cup (A'\times B')$ —— (1)

$\therefore (A\times B)' \subseteq (A'\times B) \cup (A\times B') \cup (A'\times B')$

Again, Let $(a,b) \in (A'\times B) \cup (A\times B') \cup (A'\times B')$ be arbitrary.

Then,

$(a,b) \in (A'\times B) \cup (A\times B') \cup (A'\times B')$

$\Rightarrow (a,b) \in (A'\times B) \vee (a,b) \in (A\times B') \vee (a,b) \in (A'\times B')$

$\Rightarrow (a,b) \in (A'\times B) \vee (a\in A \wedge b\in B') \vee (a\in A' \wedge b\in B')$

$\Rightarrow (a\in A' \wedge b\in B) \vee (a\in A \wedge b\in B') \vee (a\in A' \wedge b\in B')$

$\Rightarrow (a\notin A \wedge b\in B) \vee (a\in A \wedge b\notin B) \vee (a\notin A \wedge b\notin B)$

$\Rightarrow (a,b) \notin (A\times B)$

$\Rightarrow (a,b) \in (A\times B)'$

$\therefore (A'\times B) \cup (A\times B') \cup (A'\times B') \subseteq (A\times B)'$ —— (2)

Now, from (1) and (2)

$$(A\times B)' = (A'\times B) \cup (A\times B') \cup (A'\times B')$$

Proved.

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Problems based on Cartesian products.

Q.1. Show by example that

(i) $A \cup (B \times C) \neq (A \cup B) \times (A \cup C)$

Soln: Let $A = \{1\}, B = \phi, C = \{3\}$.

$\therefore B \times C = \phi \times \{3\} = \{3\}$

Then, ~~Hence~~, $A \cup (B \times C) = \{1\} \cup \{3\} = \{1, 3\}$ —— (1)

Again, $A \cup B = \{1\} \cup \phi = \{1\} \times \{\ \} = \{1\}$

And $A \cup C = \{1\} \cup \{3\} = \{1, 3\}$

$\therefore (A \cup B) \times (A \cup C) = \{1\} \times \{1, 3\}$

$= \{(1,1), (1,3)\}$ —— (2)

From (1) and (2)

$A \times (B \times C) \neq (A \cup B) \times (A \cup C)$

Proved.

(ii) Show by example that

$A \cap (B \times C) \neq (A \cap B) \times (A \cap C)$

Soln: Let $A = \{1, 2\}, B = \{1\}, C = \{2, 3\}$

$\therefore B \times C = \{1\} \times \{2, 3\} = \{(1,2), (1,3)\}$

$\therefore A \cap (B \times C) = \{1, 2\} \cap \{(1,2), (1,3)\}$

$\Rightarrow A \cap (B \times C) = \cancel{\{1,2\}} = \phi$ —— (1)

~~Also~~ $A \cap B = \{1, 2\} \cap \{1\} = \{1\}$

and $A \cap C = \{1, 2\} \cap \{2, 3\} = \{2\}$.

$\therefore (A \cap B) \times (A \cap C) = \{1\} \times \{2\} = \{(1,2)\}$ —— (2)

From (1) and (2)

$A \times (B \times C) \neq (A \cap B) \times (A \cap C)$

Proved